RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER END SEM EXAMINATION, MARCH 2021
FIRST YEAR [BATCH 2020-23]

Date : 26/03/2021 **MATHEMATICS**
Time : 11.00 am – 1.00 pm **Paper** : MACT 2 Full Marks:50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Write your answer to the point and in your own words

Group A

Answer question no. 1 and any **one** question from question no. $2-3$. [5+10 x 1 = 15 marks]

1. Discuss the nature of the conic represented by $11x^2-4xy+14y^2-58x-44y+71=0$. [5]

2. (a) Show that the equation of the circle, which passes through the focus of the parabola $\frac{2a}{r}=1+\cos\theta$ and touches it at the point $\theta=\alpha$ is given by $r\cos^3\frac{\alpha}{2}=a\cos(\theta-\frac{3\alpha}{2})$. [5]

   (b) Find the locus of poles of the tangents to the auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to the ellipse. [5]

3. (a) If PCP' and DCD' be two mutually perpendicular diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then prove that $\frac{1}{CP^2}+\frac{1}{CD^2}=\frac{1}{a^2}+\frac{1}{b^2}$. [5]

(b) Find the equation of the pair of straight lines through the point (x_0, y_0) each of which forms an isosceles triangle with the bisectors of the angles between the straight lines $a'x^2 + 2h'xy + b'y^2 = 0$. [5]

Group B

Answer any **one** question from question no. 4 – 5. [10 x 1 = 10 marks]

4. (a) Forces acting on a particle having magnitudes 4 , 3 , 2 kg wt. and act in the direction of the vectors $3\vec{i} + 2\vec{j} + \vec{k},\ \vec{i} + 3\vec{j} + 2\vec{k}$ *and* $2\vec{i} + 3\vec{j} - \vec{k}$ respectively. These forces remain constant while the particle is displaced from the point A $(3\vec{i} + \vec{j} - 5\vec{k})$ to B $(4\vec{i} - \vec{j} + \vec{k})$. Find the work done by the forces, the unit of length being 1 meter. [5]

   (b) Solve the vector equation $k\vec{r} + \vec{r} \times \vec{a} = \vec{b}$ for $\vec{r}$ where k is a non-zero scalar, $\vec{a}$ and $\vec{b}$ are two vectors. [5]

5. (a) Find the angle between the planes $\vec{r}.(2\vec{i} - 3\vec{j} - 6\vec{k}) = 21$ and $\vec{r}.(6\vec{i} + 2\vec{j} - 9\vec{k}) = 22$. Find the equations of the bisecting planes of the angle between the given planes and indicate which plane bisects the angle containing the origin. [6]

   (b) i. Show that the lines $\vec{r} = \vec{a} + t(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + s(\vec{c} \times \vec{a})$ will intersect if $\vec{a}.\vec{c} = \vec{b}.\vec{c}$. [2]

   ii. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ then explain all the different possibilities by which the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ may be related. [2]

Group C

Answer question no. 6 and any **two** questions from question no. 7 – 9. [5+10 x 2 = 25 marks]

6. Reduce the differential equation $(x^2 + y^2)(1 + p)^2 – 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$ where $p = \frac{dy}{dx}$, into Clairaut's form and hence solve it. [5]

7. (a) Using the symbolic operator, solve the differential equation $(D–1)^2(D^2 + 1)y = e^x + \sin^2 \frac{x}{2}$. $(D \equiv \frac{d}{dx})$ [5]

   (b) Using the method of variation of parameters, solve the equation $(D^2 + 2D + 5)y = e^{-x} \sec 2x$. $(D \equiv \frac{d}{dx})$ [5]

8. (a) Solve by using the method of undetermined coefficients: $(D^2 + 2D + 2)y = x^2 + \sin x$. $(D \equiv \frac{d}{dx})$ [6]

   (b) Solve : $x\frac{dy}{dx} + y = y^2x^3 \cos x$. [4]

9. (a) i. Find an integrating factor of the equation $(x^4y^2 – y)dx + (x^2y^4 – x)dy = 0$ and hence solve it. [3]

   ii. Find the orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, λ being parameter [3]

   (b) Solve : $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$. [4]

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